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LONG-TERM STRENGTH OF METALS UNDER AN EQUIAXIAL PLANE STRESS STATE

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This paper reports test results suggesting a significant dependence of the long-term strength of metals on the form of the stress state and the method of short-term loading. The experimental data obtained are described using a modification of the kinetic theory of long-term strength containing a vector damage parameter and taking into account strength anisotropy and the damage due to short-term loading. It is shown that the experimental and theoretical values of the time to rupture are in good agreement.

Key words: *anisotropy, long-term strength, metals, damage, equiaxial plane tension.*

1. Comparison of Test Results for Uniaxial and Biaxial Tension. We consider uniaxial stress state ($\sigma_1 = \sigma_0 > 0$ and $\sigma_2 = \sigma_3 = 0$) and equiaxial plane stress state ($\sigma_1 = \sigma_2 = \sigma_0 > 0$ and $\sigma_3 = 0$) at the same stress σ_0. The results of experiments [1–3] show that the time to rupture t_1^* for uniaxial tension is much larger than the time to rupture t_2^* for biaxial tension ($c = t_1^*/t_2^* > 1$).

Results of experiments with thin-walled tubular samples of Kh18N10T stainless steel at a temperature of 850°C are given in [1]. In the case $\sigma_0 = 50$ MPa, the average value for 11 tests ($t_1^* = 12$–30 h) is $t_1^* = 21.8$ h; thus $t_2^* = 8.3$ h, and, hence, the ratio $c = 2.6$. In the case $\sigma_0 = 60$ MPa, the average value for six tests ($t_1^* = 6.7$–20.5 h) is $t_1^* = 15.4$ h, and, hence, $t_2^* = 5.1$ h and $c = 3$. Results of similar tests for another series of Kh18N10T steel samples at a temperature of 850°C are given in [2]. At $\sigma_0 = 60$ MPa, the average values are $t_1^* = 10$ h and $t_2^* = 4$ h, and, hence, $c = 2.5$.

Experimental data on the long-term strength of rectangular plates of an Al–Mg–Si aluminum alloy at a temperature of 210°C are given in [3]. For various values of the stress σ_0, the ratio $c = 1.8$–3.2.

Thus, available experimental data suggest that, with the addition of a transverse tensile stress of the same value to the tensile stress, the time to rupture decreases severalfold.

2. Vector Representation of Damage. In calculations of the long-term strength of structural members under a complex stress state, it is common to use the criterial approach. This approach takes into account one characteristic of the stress state — the so-called equivalent stress σ_e, i.e., various combinations of the stress tensor components which have the mechanical meaning or the meaning of the maximum tensile stress, or the tangential stress intensity, or the difference between the maximum and minimum principal stresses, etc. Since, in uniaxial and biaxial tensions ($\sigma_1 = \sigma_2 = \sigma_0$ and $\sigma_3 = 0$), the examined equivalent stresses coincide ($\sigma_e = \sigma_0$), is impossible to obtain different values of t_1^* and t_2^* using the criterial relation $t^* = t^*(\sigma_e)$.

In [4], the vector damage characteristic $\boldsymbol{\omega}$ was introduced to describe creep damage accumulation in a metal under a complex stress state. In Cartesian coordinates 1, 2, and 3, the velocities of the projections ω_k of the vector $\boldsymbol{\omega}$ onto the directions of the principal stresses σ_k are given by the relations

$$\frac{d\omega_k}{dt} = \dot{\omega}_k = \begin{cases} f(\sigma_k, \omega_k), & \sigma_k > 0, \\ 0, & \sigma_k \leqslant 0, \end{cases} \qquad k = 1, 2, 3. \tag{1}$$

In this case, the expression for damage $\omega = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}$ should satisfy the conditions $\omega(0) = 0$ and $\omega(t^*) = 1$.

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3. Accounting for Instantaneous Damage for an Isotropic Material. In describing the dependence of the time to rupture t^* on the form of the stress state [1–3], we use a generalization of the vector approach [4] taking into account the damage accumulated during loading. As one of the possible models that provide different values of the times t_1^* and t_2^* under tensile stress, we consider the system of relations

$$d\omega_k = \frac{d\varphi(\sigma_k)}{d\sigma_k}\, d\sigma_k + f(\sigma_k)\, dt, \qquad k = 1, 2, \tag{2}$$

where the function $\varphi(\sigma_k)$ characterizes the value of the projection ω_k of the vector of the damage accumulated during loading; $f(\sigma_k)$ is the constant velocity of the projection ω_k in time t. In the case of uniaxial tension, system (2) implies that

$$\omega_1(t) = \varphi(\sigma_0) + f(\sigma_0)t, \qquad \omega_2 = 0, \qquad t_1^* = [1 - \varphi(\sigma_0)]/f(\sigma_0), \tag{3}$$

in the case of equiaxial plane tension ($\sigma_1 = \sigma_2 = \sigma_0$ and $\sigma_3 = 0$), from (2) we have

$$\omega_1(t) = \omega_2(t) = \varphi(\sigma_0) + f(\sigma_0)t, \qquad t_2^* = [\sqrt{2}/2 - \varphi(\sigma_0)]/f(\sigma_0). \tag{4}$$

From relations (3) and (4), it follows that the instantaneous value of the damage $0 < \varphi(\sigma_0) < \sqrt{2}/2$ and the ratio $c = t_1^*/t_2^* = (1 - \varphi(\sigma_0))/(\sqrt{2}/2 - \varphi(\sigma_0)) > \sqrt{2}$ for any values of σ_0 in the indicated range. Using the results of tests [3], for $\sigma_0 = 56.2$ MPa we obtain $t_1^* = 900$ h, $t_2^* = 280$ h. For $\varphi(\sigma_0) = 0.57$ and $f(\sigma_0) = 4.78 \cdot 10^{-4}$ h^{-1}, the values of t_1^* and t_2^* calculated by relations (3) and (4) coincide with the corresponding experimental values.

In the kinetic relations (2), the ratio c depends only on the damage $\varphi(\sigma_0)$ accumulated under quasistatic loading. Accounting for instantaneous damage in the form of (2) allows one to describe experimental data for $c \geqslant \sqrt{2}$. In this case, the result does not depend on the character of creep damage accumulation.

4. Accounting for Material Anisotropy and Relations between the Damage Vector Components. During manufacture of thin-walled tubes, the material strength characteristics can acquire anisotropy. For example, as a result of low-temperature thermal treatment of high-strength steel tubes [5], the ratio of the short-term rupture strength in the axial direction to the hoop rupture strength can take values of 1.25–2.50, depending on the treatment conditions.

The joint action of the internal pressure q and the additional axial force P in thin-walled tubes leads to biaxial tension ($\sigma_z > 0$, $\sigma_\theta > 0$, and $\sigma_r = 0$); therefore, according to (1), $\omega_r = 0$. Equation (1) can be written as

$$\frac{d\omega_k}{dt} = \begin{cases} G\omega_k^{-1}\omega^{2\gamma}(\hat\sigma_k)^n, & \hat\sigma_k > 0, \\ 0, & \hat\sigma_k \leqslant 0, \end{cases} \qquad k = z, \theta, \tag{5}$$

where $\hat\sigma_k$ are the values of the given principal stresses ($\hat\sigma_z = \sigma_z/\alpha$ and $\hat\sigma_\theta = \sigma_\theta$, where α is the strength anisotropy coefficient, which is the ratio σ_z/σ_θ of the stresses leading to rupture for the same time t^* [6]); G, n, and γ are constants. In the case of uniaxial tension, the kinetic equation (5) implies that

$$\frac{d\omega_z}{dt} = G\omega_z^{2\gamma-1}\Big(\frac{\sigma_z}{\alpha}\Big)^n, \qquad \omega_z = \omega, \qquad \frac{d}{dt}\Big(\frac{\omega^{2(1-\gamma)}}{2(1-\gamma)}\Big) = G\Big(\frac{\sigma_0}{\alpha}\Big)^n,$$

$$\frac{\omega^{2(1-\gamma)}}{2(1-\gamma)} = G\Big(\frac{\sigma_0}{\alpha}\Big)^n t_1, \qquad t_1^* = \frac{1}{2G(1-\gamma)(\sigma_0/\alpha)^n}. \tag{6}$$

For biaxial tension ($\sigma_1 = \sigma_2 = \sigma_0 > 0$ and $\sigma_3 = 0$), as a result of transformations and integration of Eqs. (5), we obtain

$$\frac{d\omega^2}{dt} = 2G\omega^{2\gamma}\Big(\Big(\frac{\sigma_0}{\alpha}\Big)^n + \sigma_0^n\Big), \qquad \frac{\omega^{2(1-\gamma)}}{1-\gamma} = 2G\Big(\Big(\frac{\sigma_0}{\alpha}\Big)^n + \sigma_0^n\Big)t_2,$$

$$t_2^* = \frac{1}{2G(1-\gamma)((\sigma_0/\alpha)^n + \sigma_0^n)}. \tag{7}$$

From relations (6) and (7), we have

$$c = \frac{t_1^*}{t_2^*} = \frac{(\sigma_0/\alpha)^n + \sigma_0^n}{(\sigma_0/\alpha)^n} = 1 + \alpha^n. \tag{8}$$

Formula (8) gives the single values $c = 2$ in the case of an isotropic material ($\alpha = 1$) and $c > 2$ in the case of an anisotropic material ($\alpha > 1$). We note that the ratio c depends only on the values of α and n and does not depend on the other constants and the stress state σ_0.

5. Effect of Short-Term Loading Trajectory on Long-Term Strength Taking into Account Material Anisotropy. The effect of the method of short-term loading on the time to rupture for constant stress-tensor components was studied at the Laboratory of Creep and Long-Term Strength of Metals, Institute of Mechanics, Moscow State University. The tests were performed as follows [2]. Thin-walled samples of Kh18N10T stainless steel at a temperature of 850°C were tested for long-term strength under combined action of tensile and internal pressures. After loading, the principal stresses σ_z and σ_θ became equal: $\sigma_z = \sigma_\theta = \sigma_0 = 60$ 10). In different samples, the specified stress state was achieved using three methods (programs) of short-term loading. Two samples were first subjected to internal pressure at which $\sigma_z = 30$ MPa and $\sigma_\theta = 60$ MPa, and then to additional loading by a tensile force, during which the axial stress σ_z reached 60 MPa; the times to rupture in the two samples were 6.0 and 6.1 h, respectively. The other two samples were first loaded by axial tension and then by additional internal pressure; the times to rupture were 2 and 3 h, respectively. According to the third loading program, small increments of the tensile force P and internal pressure q were added alternately; in this case, the samples ruptured in times of 3.4 and 3.8 h. In all six tests, the loading time was approximately 3 min which is on the average two orders of magnitude smaller than the time of the subsequent tests under constant stress. Despite the small number of tests, the experimental studies suggest a significant dependence of the long-term strength on the short-term loading program. In describing this dependence, we assume that the material damage ω is accumulated from the beginning of application of tensile stress, i.e., by the end of the loading program ($\sigma_z = \sigma_\theta = \sigma_0 = 60$ MPa), the material damage takes the initial value $\omega_0 = \omega(+0)$. Next, the damage ω develops under creep conditions to the value $\omega(t^*) = 1$ corresponding to the moment of rupture.

The increase in the component ω_k of the vector $\boldsymbol{\omega}$ in the tested tubular samples is described by the kinetic equations

$$d\omega_k = \varphi(s_k, \omega_k, \omega)\, d\sigma_k + f(s_k, \omega_k, \omega)\, dt, \qquad k = z, \theta \tag{9}$$

(s_k are the components of the stress deviator σ_k in the principal axes).

For the different short-term loading programs, we find the values of ω_0. To describe short-term loading using system (9), we introduce strength anisotropy of the components ω_z and ω_θ of the vector $\boldsymbol{\omega}$, i.e., we assume that the equality $\omega_z = \omega_\theta$ is satisfied if the components of the stress deviator satisfy the relation $s_\theta = s_z/\alpha$ ($\alpha \geqslant 1$). We introduce the field of the given stress deviator component: $\hat{s}_z = s_z/\alpha$ and $\hat{s}_\theta = s_\theta$. Then, the dependence of the component ω_k on the given stress deviator components $\hat{s}_k$ becomes isotropic. The kinetic equations (9) is written as

$$d\omega_k = \begin{cases} (Q/(\sigma_{00})^{m+1})\omega_k^{-1}\omega^{2\beta}(\hat{s}_k)^m\, d\hat{\sigma}_k, & \hat{s}_k > 0, \\ 0, & \hat{s}_k \leqslant 0, \end{cases} \qquad k = z, \theta,$$

$$\omega = \sqrt{\omega_z^2 + \omega_\theta^2}, \qquad \omega(-0) = 0, \quad \omega(+0) = \omega_0, \quad \omega(t^*) = 1, \tag{10}$$

where σ_{00} is an arbitrary quantity which has the dimension of stress, $\hat{s}_z$ and $\hat{s}_\theta$ are the components of the given stress deviator, Q, m, and β are constant dimensionless quantities. Since the radial stress $\hat{\sigma}_r \equiv 0$, we have the average stress $\hat{\sigma} = (\hat{\sigma}_z + \hat{\sigma}_\theta + \hat{\sigma}_r)/3 = (\hat{\sigma}_z + \hat{\sigma}_\theta)/3 = (\sigma_z/\alpha + \sigma_\theta)/3$. In this case, the deviators of the given stresses are expressed as

$$\hat{s}_z = \hat{\sigma}_z - \hat{\sigma} = (2\sigma_z - \alpha\sigma_\theta)/(3\alpha), \qquad \hat{s}_\theta = \hat{\sigma}_\theta - \hat{\sigma} = (2\alpha\sigma_\theta - \sigma_z)/(3\alpha). \tag{11}$$

According to (10), accumulation of the component ω_k occurs only for positive values of the stress deviator components $\hat{s}_z$ and $\hat{s}_\theta$. Substituting formulas (11) into (10), for $\hat{s}_z > 0$ and $\hat{s}_\theta > 0$, we obtain

$$2\omega_z\omega^{-2\beta}\, d\omega_z = (\omega_z^2 + \omega_\theta^2)^{-\beta}\, d\omega_z^2 = \frac{2Q}{(\sigma_{00})^{m+1}}\,(\hat{s}_z)^m\, d\hat{\sigma}_z = \frac{2Q}{(\sigma_{00})^{m+1}}\Big(\frac{2\sigma_z - \alpha\sigma_\theta}{3\alpha}\Big)^m \Big(\frac{1}{\alpha}\Big)\, d\sigma_z,$$

$$2\omega_\theta\omega^{-2\beta}\, d\omega_\theta = (\omega_z^2 + \omega_\theta^2)^{-\beta}\, d\omega_\theta^2 = \frac{2Q}{(\sigma_{00})^{m+1}}\,(\hat{s}_\theta)^m\, d\hat{\sigma}_\theta = \frac{2Q}{(\sigma_{00})^{m+1}}\Big(\frac{2\alpha\sigma_\theta - \sigma_z}{3\alpha}\Big)^m\, d\sigma_\theta. \tag{12}$$

Let us introduce new components of the damage vector and dimensionless stresses: $\Omega_k = (2Q)^{0.5(\beta-1)}\omega_k$ and $\bar{\sigma}_k = \sigma_k/\sigma_{00}$. Below, the bar above the dimensionless stresses is omitted and Eqs. (12) become

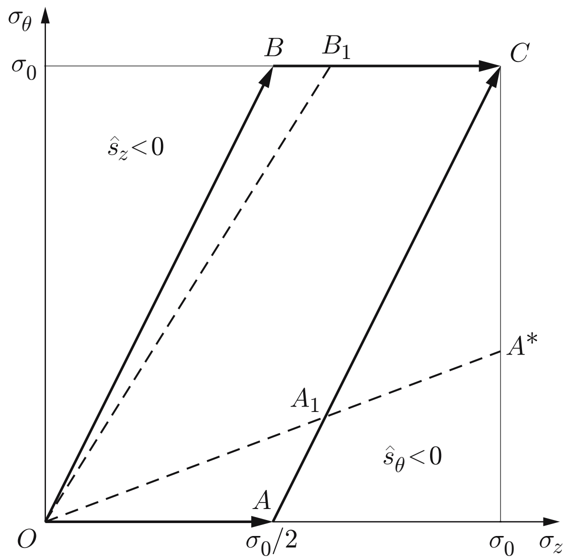

Fig. 1. Diagram of loading of tubular samples to the stress $\sigma_z = \sigma_\theta = \sigma_0$ along a two-link trajectory (the dashed lines are the boundaries of the regions $\hat{s}_z < 0$ and $\hat{s}_\theta < 0$).

$$(\Omega_z^2 + \Omega_\theta^2)^{-\beta}\, d\Omega_z^2 = \frac{1}{\alpha}\Big(\frac{2\sigma_z - \alpha\sigma_\theta}{3\alpha}\Big)^m d\sigma_z, \quad (\Omega_z^2 + \Omega_\theta^2)^{-\beta}\, d\Omega_\theta^2 = \Big(\frac{2\alpha\sigma_\theta - \sigma_z}{3\alpha}\Big)^m d\sigma_\theta. \tag{13}$$

It should be noted that system (13) characterizes the variation in the quantity Ω only for positive values of the deviator components of the given stresses. Hence, the following inequalities are valid:

$$\hat{s}_z > 0, \quad 2\sigma_z - \alpha\sigma_\theta > 0, \qquad \hat{s}_\theta > 0, \quad 2\alpha\sigma_\theta - \sigma_z > 0. \tag{14}$$

We study system (13) for loading of tubular samples on the trajectories OAC and OBC (Fig. 1) in the range from the unloaded state ($\sigma_z = \sigma_\theta = 0$) to the two-axial tensions at the point C ($\sigma_z = \sigma_\theta = \sigma_0$). At the points A and B, the stresses σ_z and σ_θ are equal to $(\sigma_z)_A = (\sigma_z)_B = \sigma_0/2$, $(\sigma_\theta)_A = 0$, and $(\sigma_\theta)_B = \sigma_0$.

We examine the loading trajectory OAC which is a two-link broken line (see Fig. 1). Along this broken line ($\hat{s}_z \geqslant 0$), the component Ω_z increases, and, along the segment OA, the stress $\hat{s}_\theta < 0$; therefore, $d\sigma_\theta = 0$. Thus, during loading from the point O to the point A, the component $\Omega_\theta = 0$. On the segment AC, we distinguish a point A_1 that separates the regions $\hat{s}_\theta < 0$ (on the segment AA_1) and $\hat{s}_\theta > 0$ (on the segment A_1C), hence $(\hat{s}_\theta)_{A_1} = 0$. At the point A_1, the stresses σ_z and σ_θ are equal to $(\sigma_z)_{A_1} = 2\alpha\sigma_0/(4\alpha - 1)$ and $(\sigma_\theta)_{A_1} = \sigma_0/(4\alpha - 1)$. Thus, the component Ω_θ increases from zero only on the segment A_1C. At the point C, we calculate the stress Ω_{OAC} accumulated during loading along the broken line OAC. Integration of the first equation of system (13) yields

$$\int\limits_0^{(\Omega_z)_{A_1}} \Omega_z^{-2\beta}\, d\Omega_z^2 = \frac{2^m}{\alpha(3\alpha)^m} \int\limits_0^{\sigma_0/2} \sigma_z^m\, d\sigma_z + \frac{1}{\alpha(3\alpha)^m} \int\limits_{\sigma_0/2}^{2\alpha\sigma_0/(4\alpha-1)} (2(1-\alpha)\sigma_z + \alpha\sigma_0)^m\, d\sigma_z. \tag{15}$$

From equality (15), we have

$$(\Omega_z^2)_{A_1} = \Big[\frac{(1-\beta)\sigma_0^{m+1}}{2(m+1)\alpha(1-\alpha)(3\alpha)^m}\Big(\Big(\frac{3\alpha}{4\alpha-1}\Big)^{m+1} - \alpha\Big)\Big]^{1/(1-\beta)}.$$

At the point C for $\sigma_z = \sigma_0$, the projections $(\Omega_z)_C$ and $(\Omega_\theta)_C$ of the stress Ω_{OAC} are found by solving system (13) with the initial conditions $\Omega_z = (\Omega_z)_{A_1}$ and $\Omega_\theta = 0$; as a result, we obtain $\Omega_{OAC}^2 = (\Omega_z^2)_C + (\Omega_\theta^2)_C$.

Let us consider the loading trajectory OBC, which is also a two-link broken line (see Fig. 1). Because, on the segment OB, $\hat{s}_z < 0$, it follows that $\Omega_z = 0$. On the segment BC, we distinguish a point B_1 which satisfies the condition $(\hat{s}_z)_{B_1} = 0$. Then, $(\sigma_z)_{B_1} = 0.5\alpha\sigma_0$ and $(\sigma_\theta)_{B_1} = \sigma_0$. On the segment BB_1, $\hat{s}_z < 0$; hence, $(\Omega_z)_{B_1} = 0$. Thus, for loading along the broken line OBC, the increment $d\Omega_z \neq 0$ occurs only on the segment B_1C ($\hat{s}_z > 0$); the component Ω_θ increases from zero only on the segment OB.

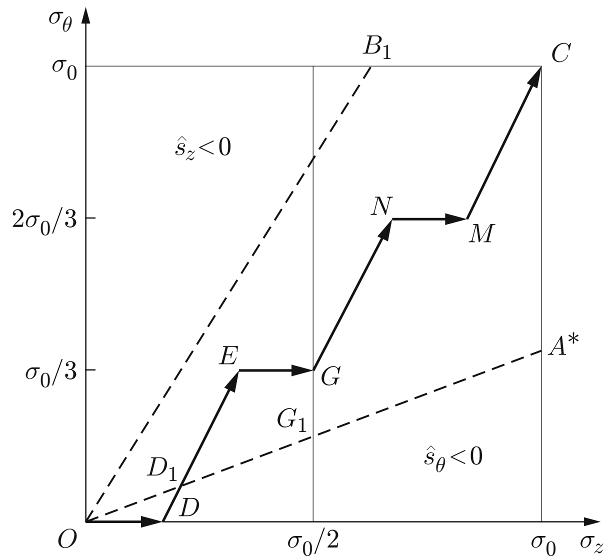

Fig. 2. Diagram of loading of tubular samples to a stress $\sigma_z = \sigma_\theta = \sigma_0$ along a multi-link trajectory (the dashed lines are the boundaries of the regions $\hat{s}_z < 0$ and $\hat{s}_\theta < 0$).

At the point C, we calculate the value of Ω_{OBC} accumulated during loading along the broken line OBC. Integrating the second equation of system (13) and taking into account that $\sigma_\theta = 2\sigma_z$ on the segment OB, we have

$$(\Omega_\theta^2)_B = \Big[\Big(\frac{1-\beta}{m+1}\Big)\Big(\frac{4\alpha-1}{6\alpha}\Big)^m\Big(\frac{\sigma_0}{2}\Big)^{m+1}\Big]^{1/(1-\beta)}. \tag{16}$$

Integrating the first equation of system (13) for motion along the segment B_1C, we obtain

$$(\Omega_z^2)_C = \Big(\frac{(1-\beta)((2-\alpha)\sigma_0)^{m+1}}{2(m+1)\alpha(3\alpha)^m} + (\Omega_\theta^{2(1-\beta)})_B\Big)^{1/(1-\beta)} - (\Omega_\theta^2)_B. \tag{17}$$

Taking into account Eqs. (16) and, (17), we have $(\Omega^2)_{OBC} = (\Omega_z^2)_C + (\Omega_\theta^2)_B$.

Let us study the damage accumulation process for six stages of sequential increases in the axial force and internal pressure with identical increments $d\sigma_z = \sigma_0/6$ in each stage of the trajectory $ODEGNMC$, which is a multilink broken line (Fig. 2). Previously, in the plane $(\sigma_z, \sigma_\theta)$ we determined the regions in which the corresponding deviators (14) are nonnegtaive. The inequality $2\sigma_z - \alpha\sigma_\theta \geqslant 0$ corresponds to the region on the right of the straight line OB_1 [the point B_1 has the coordinates $(\sigma_z)_{B_1} = 0.5\alpha\sigma_0$ and $(\sigma_\theta)_{B_1} = \sigma_0$] which does not limit damage accumulation on the loading trajectory considered. The inequality $2\alpha\sigma_\theta - \sigma_z \geqslant 0$ corresponds to the region above the straight line OA^* [the point A^* has the coordinates $(\sigma_z)_{A^*} = \sigma_0$ and $(\sigma_\theta)_{A^*} = 0.5\sigma_0/\alpha$] which intersects the straight line segment DE at the point D_1. We find the coordinates of the point D_1: on the straight line segment DE, we have $\sigma_\theta = 2\sigma_z - \sigma_0/3$, and at the point D_1, we have $\hat{s}_\theta = [2\alpha(2\sigma_z - \sigma_0/3) - \sigma_z]/(3\alpha) = 0$; hence, $(\sigma_z)_{D_1} = 2\alpha\sigma_0/(3(4\alpha-1))$ and $(\sigma_\theta)_{D_1} = \sigma_0/(3(4\alpha-1))$.

Let us prove that D_1 is the single point of intersection of the straight line OA^* and the broken line $ODEGNMC$. For this, we determine the coordinates of the point G_1 of intersection of the straight lines OA^* and $\sigma_z = \sigma_0/2$. The equation of the straight line OA^* is written as $\sigma_z = 2\alpha\sigma_\theta$. The ordinates of the points G_1 and G are equal to $(\sigma_\theta)_{G_1} = \sigma_0/4\alpha$ and $(\sigma_\theta)_G = \sigma_0/3$, respectively. The uniqueness of the point D_1 follows from the inequality $(\sigma_\theta)_G > (\sigma_\theta)_{G_1}$ for $\alpha > 3/4$. Since the anisotropy coefficient $\alpha \geqslant 1$, the uniqueness of the point D_1 is proved. Along the broken line ODD_1, we have $\Omega_\theta = 0$. Integration of the first equation of system (13) yields

$$(\Omega_z^2)_{D_1} = \Big[\frac{(1-\beta)\sigma_0^{m+1}}{2(m+1)3^m\alpha^{m+1}}\Big(1 + \frac{1}{1-\alpha}\Big(\Big(\frac{\alpha}{3(4\alpha-1)}\Big)^{m+1} - \Big(\frac{1}{3}\Big)^{m+1}\Big)\Big)\Big]^{1/(1-\beta)}.$$

On the segment D_1E, the increments $d\Omega_z$ and $d\Omega_\theta$ are not equal to zero. At the point E for $\sigma_z = \sigma_0/3$, the projections $(\Omega_z)_E$ and $(\Omega_\theta)_E$ are determined by solving (13) with the initial conditions $\Omega_z = (\Omega_z)_{D_1}$ and $(\Omega_\theta)_{D_1} = 0$.

On the segment EG, we have $d\Omega_z > 0$, $d\Omega_\theta = 0$, and $(\Omega_\theta)_G = (\Omega_\theta)_E$. Integration of the first differential equation of system (13) yields

$$(\Omega_z^2)_G = \Big(\frac{(1-\beta)\sigma_0^{m+1}}{2\alpha(m+1)(3\alpha)^m 3^{m+1}} \Big((3-\alpha)^{m+1} - (2-\alpha)^{m+1}\Big)$$

$$+ \Big((\Omega_z^2)_E + (\Omega_\theta^2)_E\Big)^{1-\beta}\Big)^{1/(1-\beta)} - (\Omega_\theta^2)_E.$$

At the point N for $\sigma_z = 2\sigma_0/3$, from the solution of (13) with the initial conditions $\Omega_z = (\Omega_z)_G$ and $\Omega_\theta = (\Omega_\theta)_E$, we find the values of the quantities $(\Omega_z)_N$ and$(\Omega_\theta)_N$, taking into account that, $\sigma_\theta = 2\sigma_z - 2\sigma_0/3$ on the segment GN. On the segment NM, we have $d\Omega_z > 0$, $d\Omega_\theta = 0$, and $(\Omega_\theta)_M = (\Omega_\theta)_N$. Integration of the first equation in (13) yields

$$(\Omega_z^2)_M = \Big(\frac{(1-\beta)\sigma_0^{m+1}}{2\alpha(m+1)(3\alpha)^m 3^{m+1}} \Big((5-2\alpha)^{m+1} - (4-2\alpha)^{m+1}\Big)$$

$$+ \Big((\Omega_z^2)_N + (\Omega_\theta^2)_N\Big)^{1-\beta}\Big)^{1/(1-\beta)} - (\Omega_\theta^2)_N.$$

On the segment MC, the equality $\sigma_\theta = 2\sigma_z - \sigma_0$ is satisfied. Solution of (13) subject to the initial condition $\Omega_z = (\Omega_z)_M$ yields the stresses $(\Omega_z)_C$ and $(\Omega_\theta)_C$ accumulated along the broken line $ODEGNMC$. At the end of loading on the trajectory $ODEGNMC$, the stress takes the value $\Omega = \sqrt{(\Omega_z^2)_C + (\Omega_\theta^2)_C}$.

For $\alpha = 1.21$, $m = 3$, $\beta = 0.3$, and $Q = 4686$, the moduli of the vector of the damage ω accumulated during loading from the point O to the point C along different trajectories are equal to

$$\omega_{OAC} = 10^{-1}, \qquad \omega_{ODEGNMC} = 8.9 \cdot 10^{-2}, \qquad \omega_{OBC} = 7.5 \cdot 10^{-3}. \tag{18}$$

Let us determine the increase in the components ω_z and ω_θ of the vector $\boldsymbol{\omega}$ during material creep at the stresses $\sigma_z = \sigma_\theta = \sigma_0$ before the moment of rupture ($t = t^*$). For this value of the parameter ω (18), as the initial values we use $\omega_0 = \omega\Big|_{t=0}$. The second term in Eq. (9) is written as

$$d\omega_k = \begin{cases} G\omega_k^{-1}\omega^{2\gamma}(\hat{s}_k)^n\,dt, & \hat{s}_k > 0, \\ 0, & \hat{s}_k \leqslant 0, \end{cases} \qquad k = z, \theta, \tag{19}$$

where G, n, and γ are constants. Under biaxial tension conditions, ($\sigma_z = \sigma_\theta = \sigma_0$), the expressions for the deviator components of the given stresses become $\hat{s}_z = (2-\alpha)\sigma_0/(3\alpha)$ and $\hat{s}_\theta = (2\alpha - 1)\sigma_0/(3\alpha)$. Combining the right and left sides of Eqs. (19) leads to

$$\omega^{-2\gamma}\,d\omega^2 = 2G(\hat{s}_z^n + \hat{s}_\theta^n)\,dt, \qquad \omega\Big|_{t=0} = \omega_0, \qquad \omega(t^*) = 1. \tag{20}$$

Integration of equality (20) yields the following dependence of the time to rupture t^* on the damage ω_0:

$$t^* = \frac{(3\alpha)^n[1 - \omega_0^{2(1-\gamma)}]}{2G(1-\gamma)\sigma_0^n[(2-\alpha)^n + (2\alpha-1)^n]}. \tag{21}$$

For $\sigma_0 = 60$ MPa, $\alpha = 1.21$, $G = 9.65 \cdot 10^{-4}$ (h$\cdot$ MPa^n)$^{-1}$, $\gamma = 0.99$, and $n = 2.04$, taking into account (18), from Eq. (21) we obtain the following values for the time to rupture: $t^*_{OAC} = 2.9$ h, $t^*_{ODEGNMC} = 3.1$ h, and $t^*_{OBC} = 6.1$ h. These values are in good agreement with the average times to rupture obtained in experiments [2]: $t^*_{OAC} = 2.5$ h, $t^*_{ODEGNMC} = 3.6$ h, and $t^*_{OBC} = 6.1$ h.

Conclusions. The proposed kinetic equations containing the vector damage parameter provide good agreement between the experimental and theoretical times to rupture for biaxial tension ($\sigma_1 = \sigma_2 = \sigma_0 > 0$ and $\sigma_3 = 0$). For the first time, the equations of damage accumulation take into account the material strength anisotropy and the damage due to short-term loading.

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REFERENCES

1. A. M. Lokoshchenko, E. A. Myakotin, and S. A. Shesterikov, "Creep and long-term strength of Kh18N10T steel under a complex stress state," *Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela*, No. 4, 87–94 (1979).
2. A. M. Lokoshchenko, "Investigation of long-term strength in a complex stress state using a kinetic approach," in: *Trans. of the Polzunov Central Research and Design Boiler-and-Turbine Inst.*, No. 230: *Problems of Long-Term Strength of Power Equipment* (1986), pp. 107–109.
3. D. R. Hayhurst "Creep rupture under multi-axial states of stress," *J. Mech. Phys. Solids*, **20**, No. 6, 381–390 (1972).
4. I. V. Nametsnikova and S. A. Shesterikov, "Vector representation of the damage parameter," in: *Deformation and Rupture of Solids* [in Russian], Izd. Mosk. Univ., Moscow (1985), pp. 43–52.
5. N. I. Chernyak, R. P. Radchenko, D. A. Gavrilov, et al., "Effect of the type and degree of plastic deformation on the mechanical properties of high-strength tubes under low-temperature thermomechanical treatment," *Probl. Prochn.*, No. 4, 51–54 (1976).
6. A. M. Lokoshchenko, "Determining anisotropy in studies of long-term strength under a plane stress state," *Probl. Prochn.*, **9**, 71–73 (1983).